



# Folding with Fivefold Geometry

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Ann Arbor, Michigan, USA

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# Folding with Fivefold Geometry

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# Background

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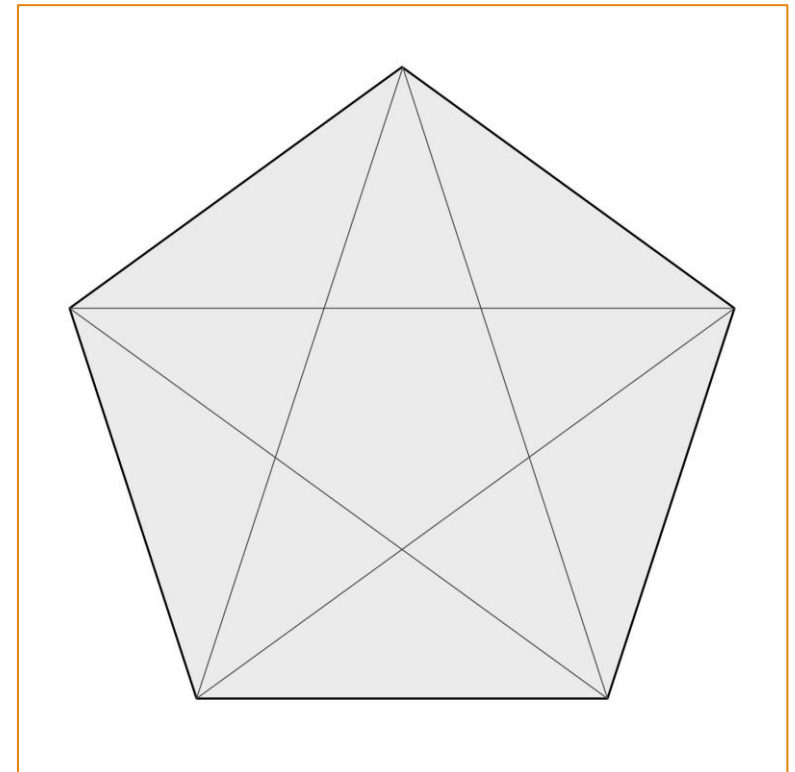
## Why Fivefold Symmetry?

- It's fun and challenging, and can lead to new and unexpected discoveries

## Folding + Geometric Principles

- Golden ratio galore
- Pentagons cannot tile a plane. You can't actually make a grid of pentagons
- But you get some really cool triangles and rhombi

Fivefold geometry is very hard – but very fun!

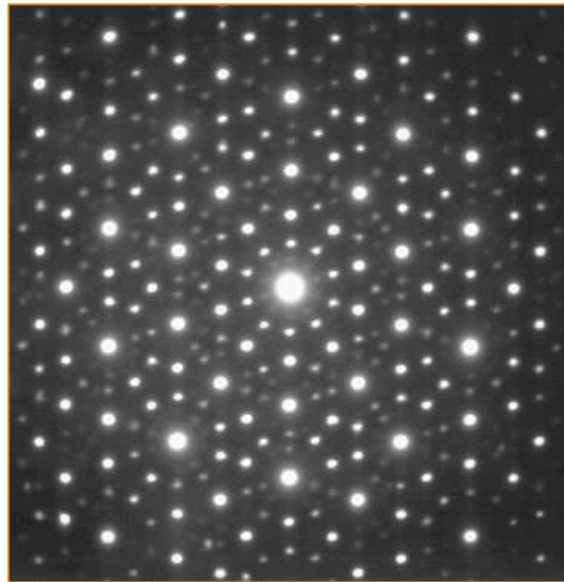


# Fivefold Symmetry in Nature

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Plants – the golden ratio is common in plant geometry, giving rise to fivefold structures

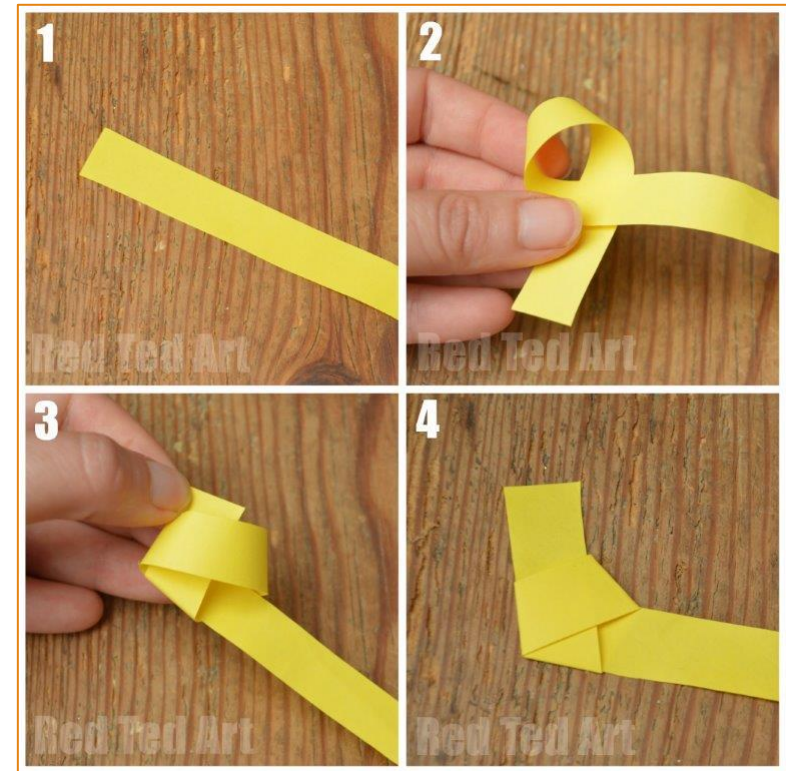
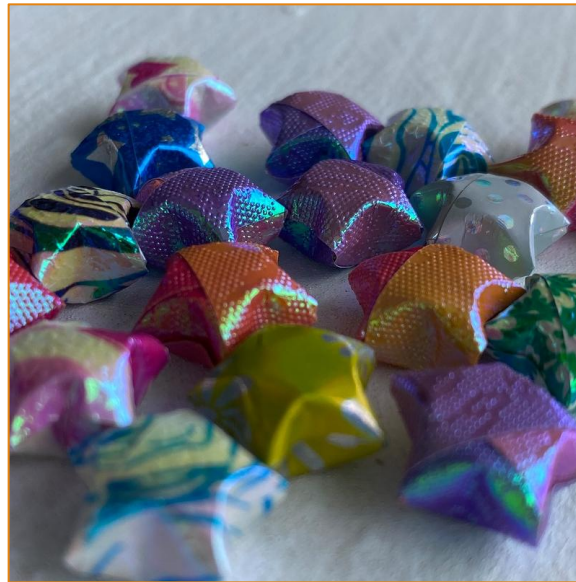
Quasicrystals - minerals



# How to Fold a Pentagon

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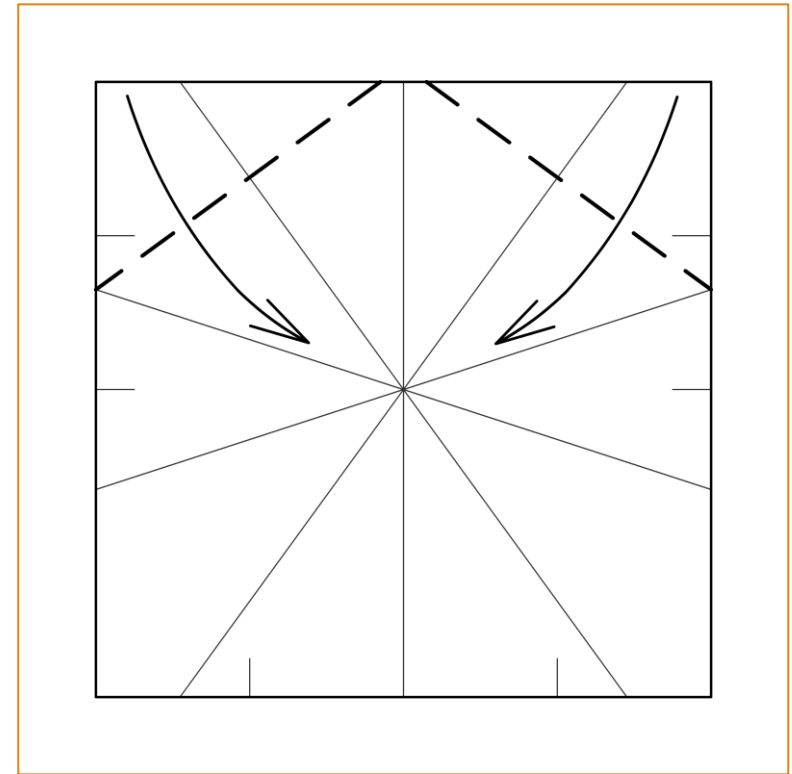
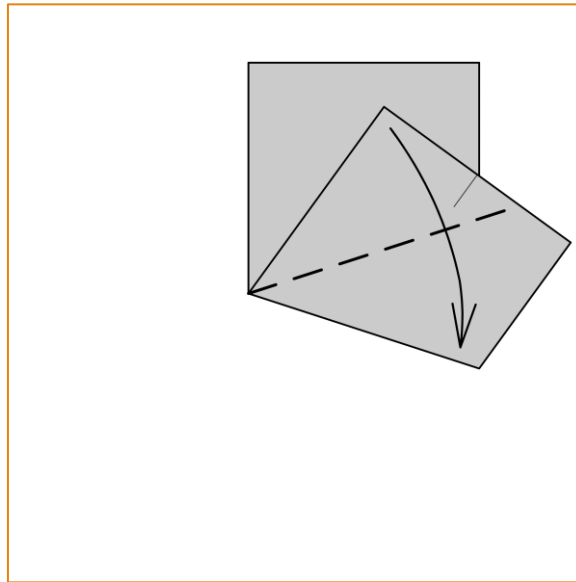
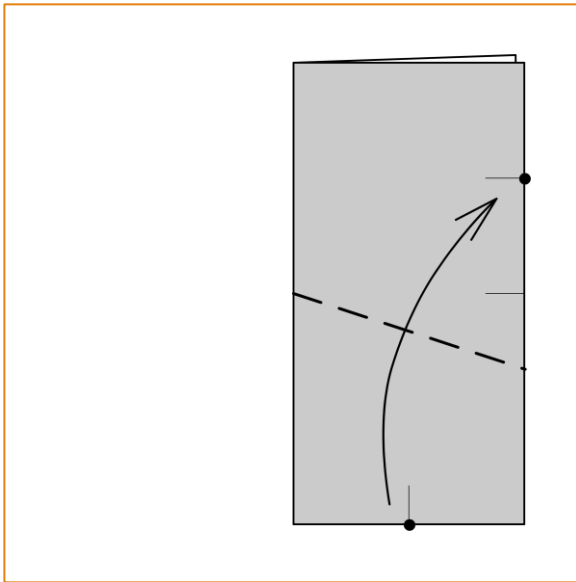
Making a pentagon from a strip (Puffy Stars)



# How to Fold a Pentagon from a Square

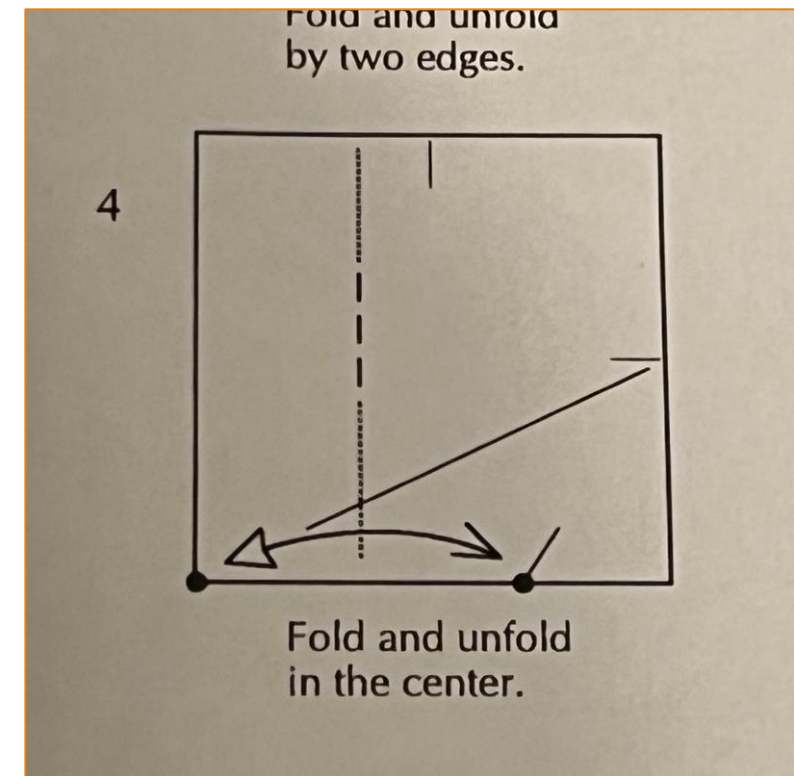
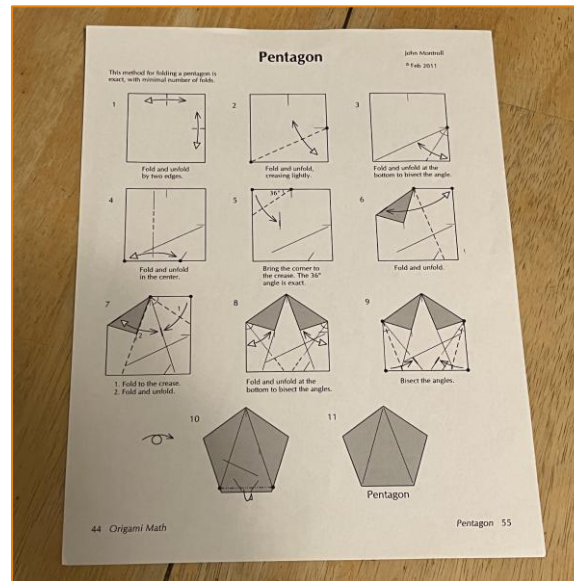
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A very good approximation, based on the fact that the slope of an 18 degree angle is very close to 3:1.



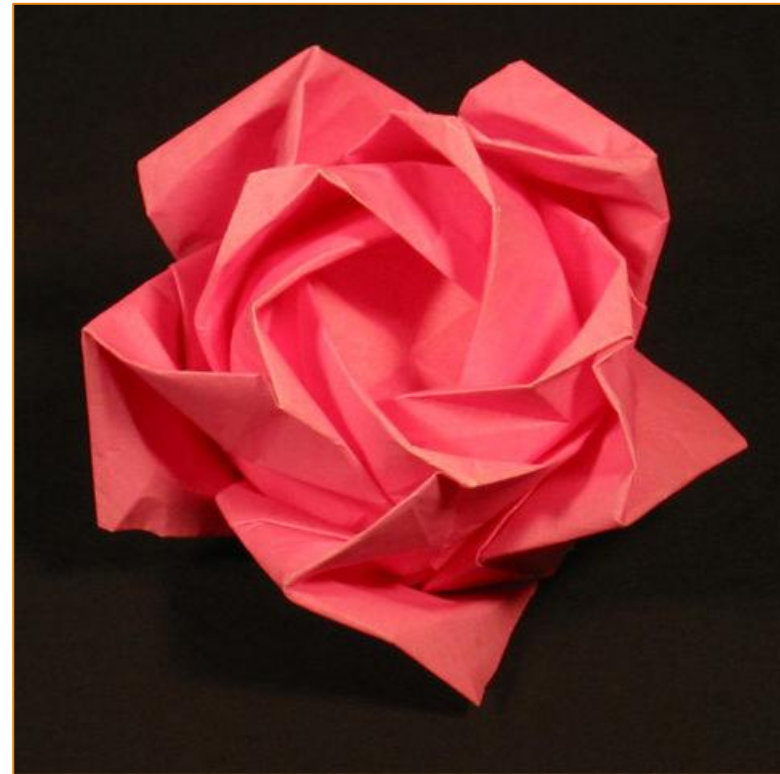
# How to Fold a Pentagon from a Square

John Montroll has a mathematically perfect method based on first finding the golden mean latent in a square.



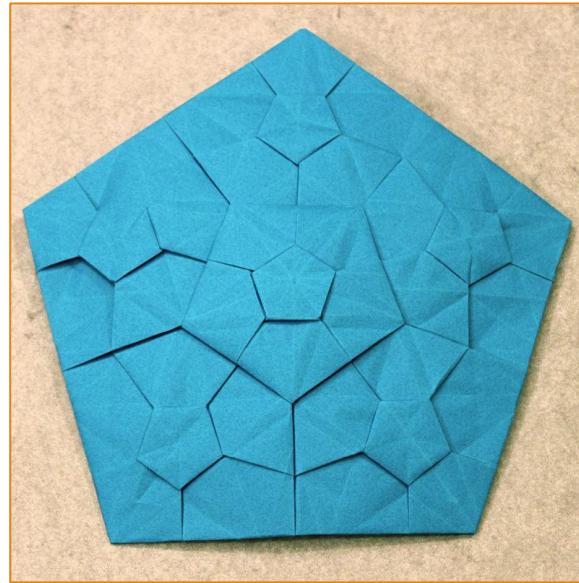
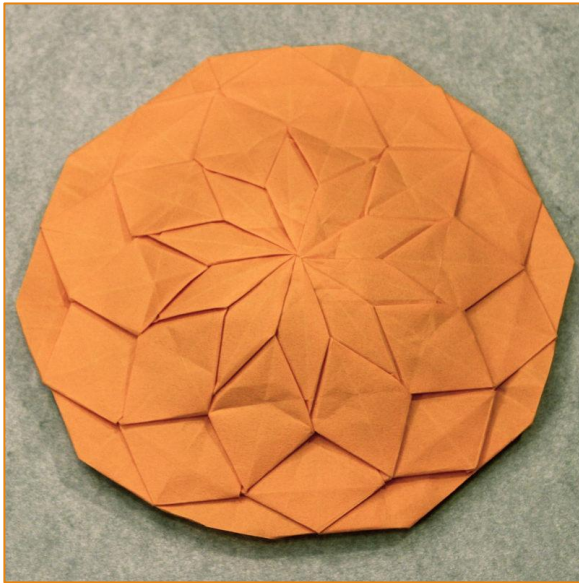
# Models – Plants and Flowers

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# Models - Tessellations

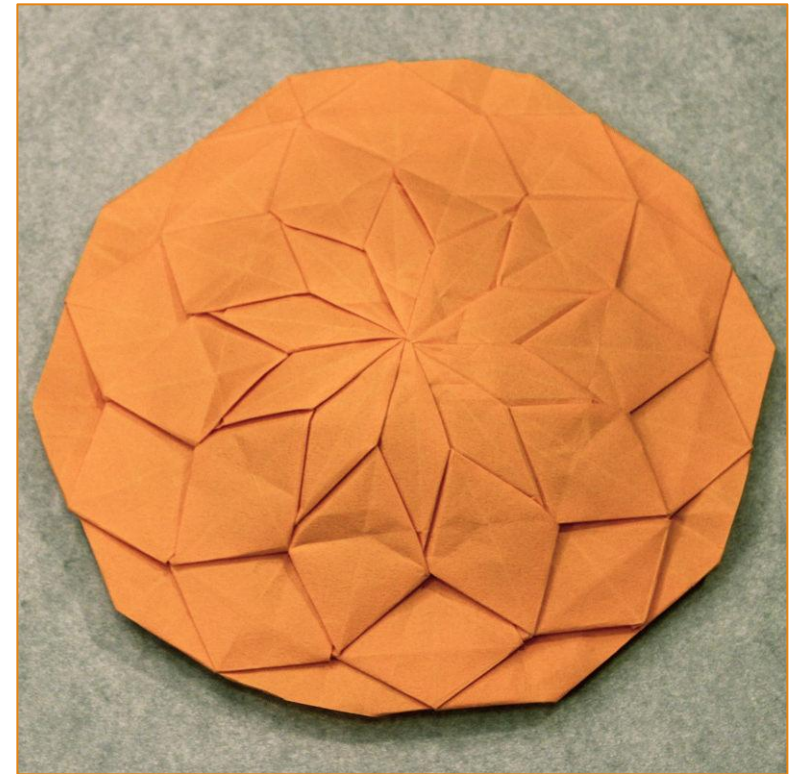
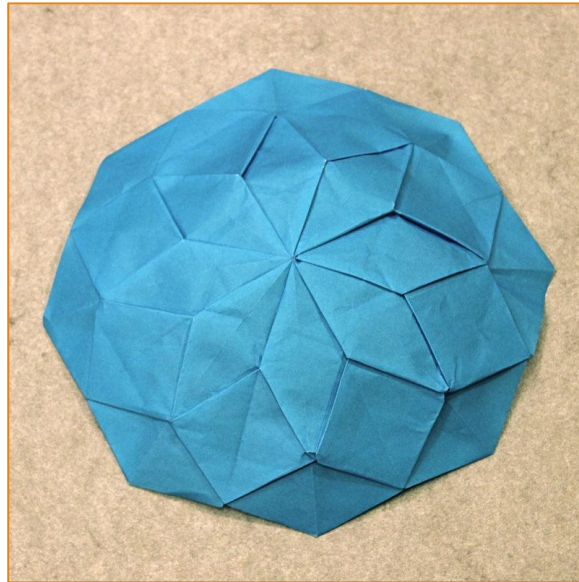
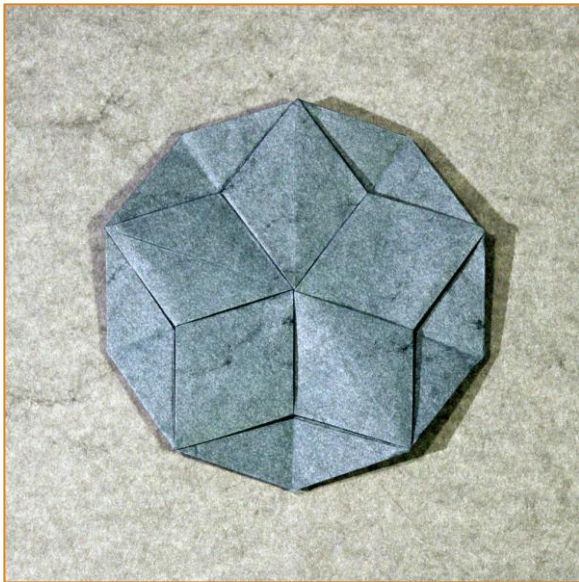
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# Mandala Tessellations

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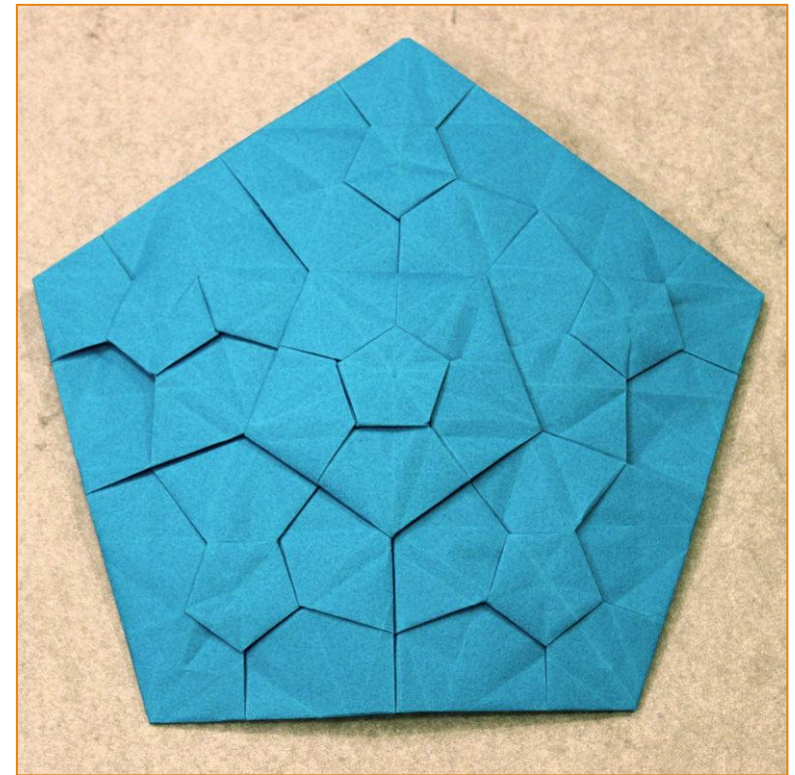
Feature polar symmetry and an array of progressively evolving parallelograms.



# Fractal Tessellations

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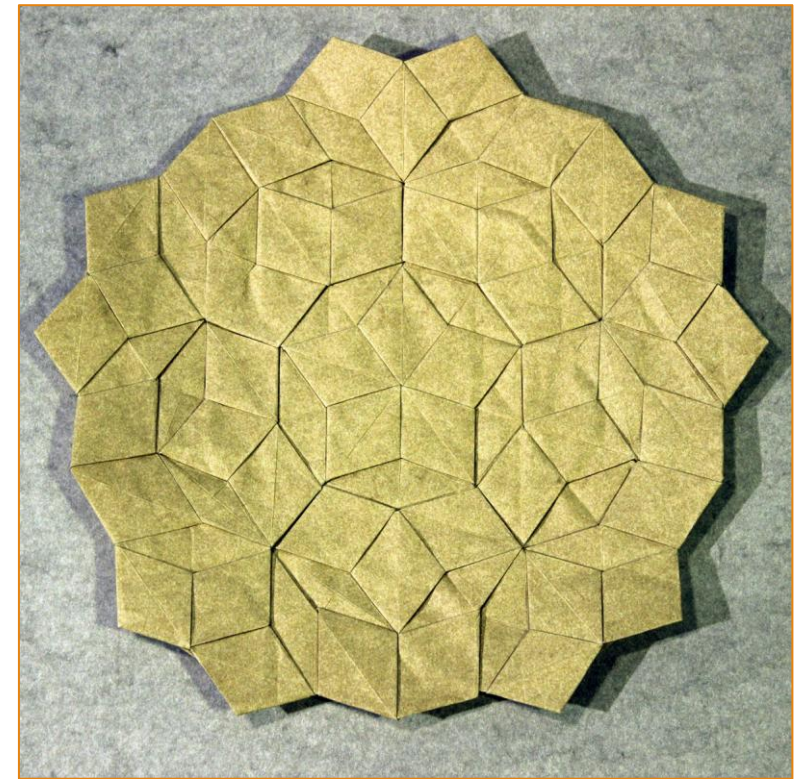
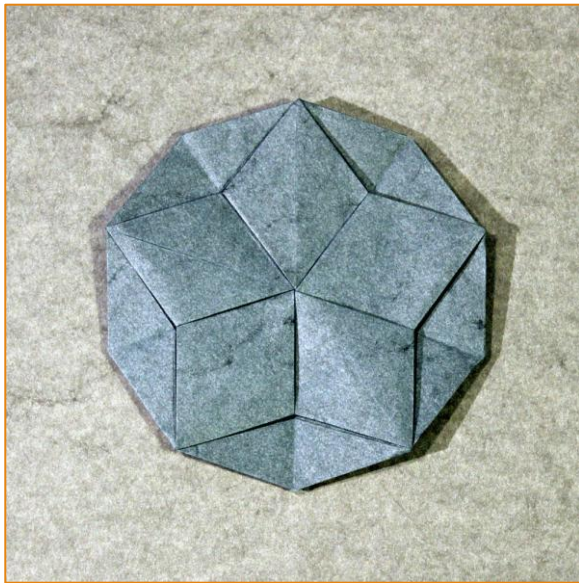
Made by subdividing the cells of of a grid into smaller cells.



# Penrose Tilings

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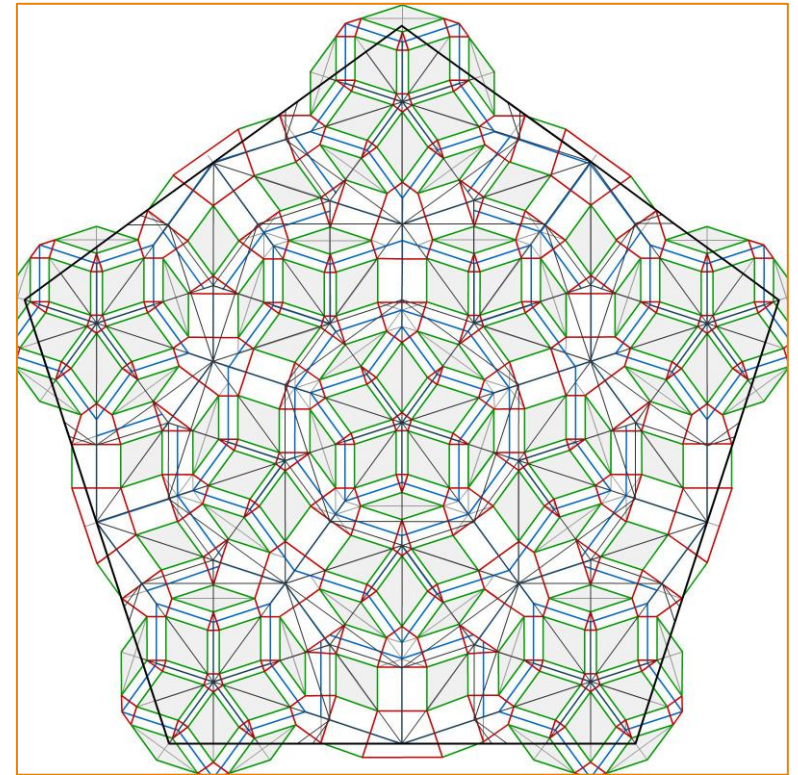
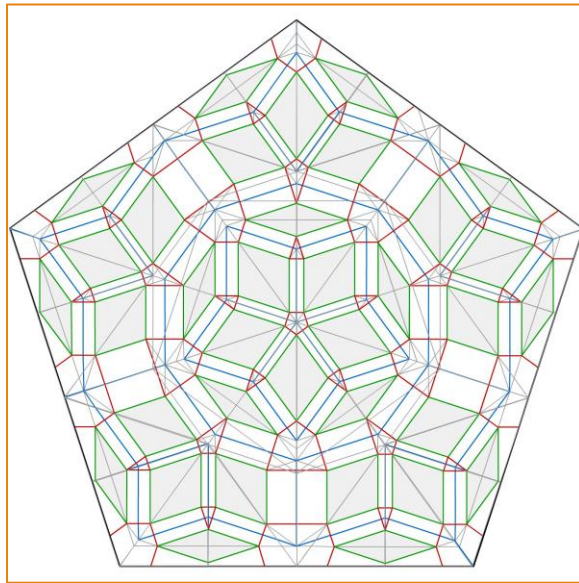
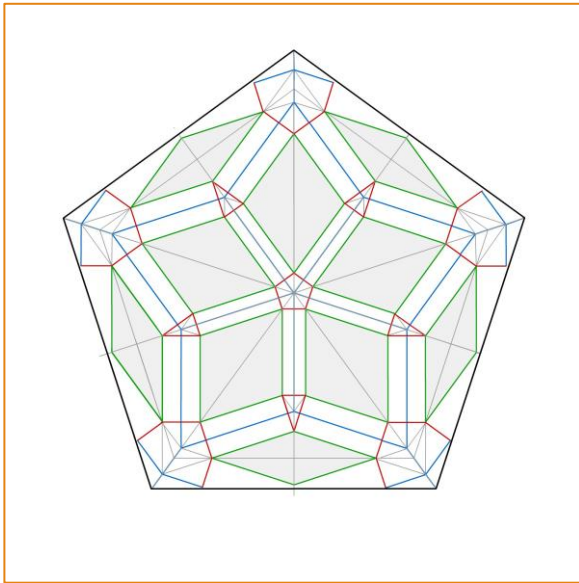
Tiling use two rhombi, defined by local joining rules. The first three iterations shown, this could go on to infinity!



# Penrose Tilings – Crease Pattern

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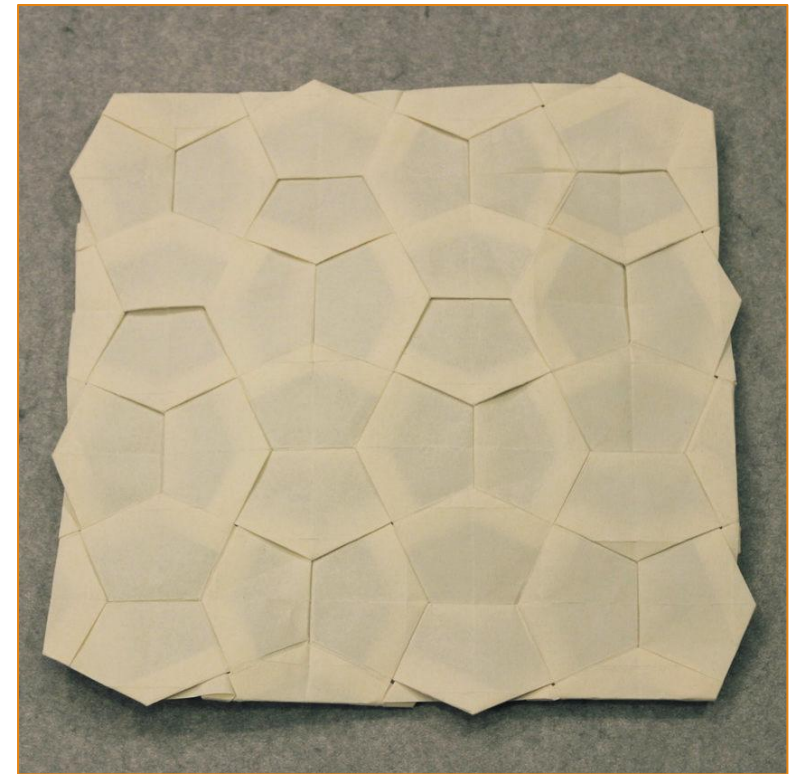
The width of the gusset between the cells is dependent on the angles where they join at the vertex.



# Other Tessellations

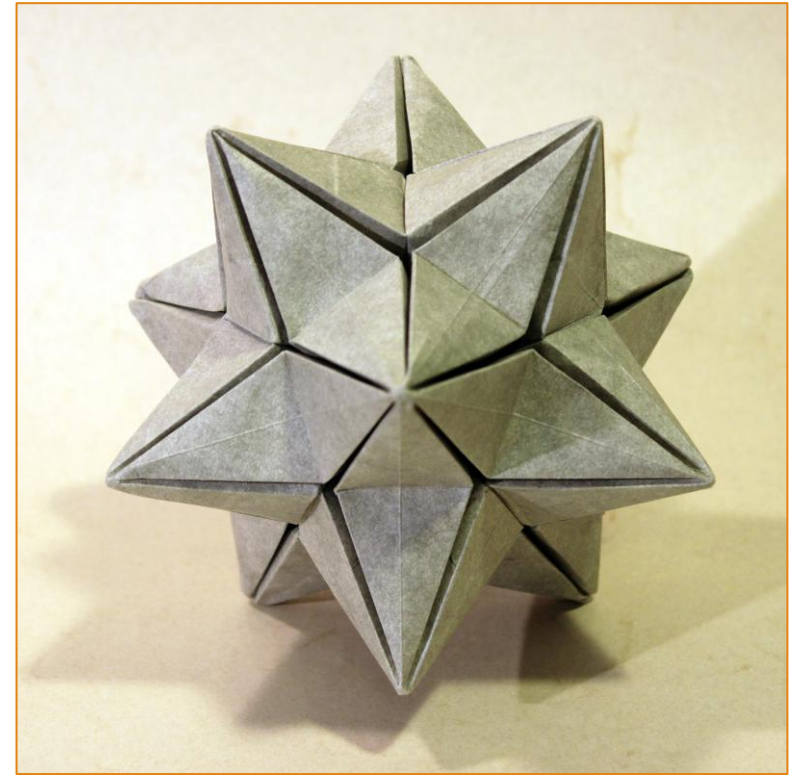
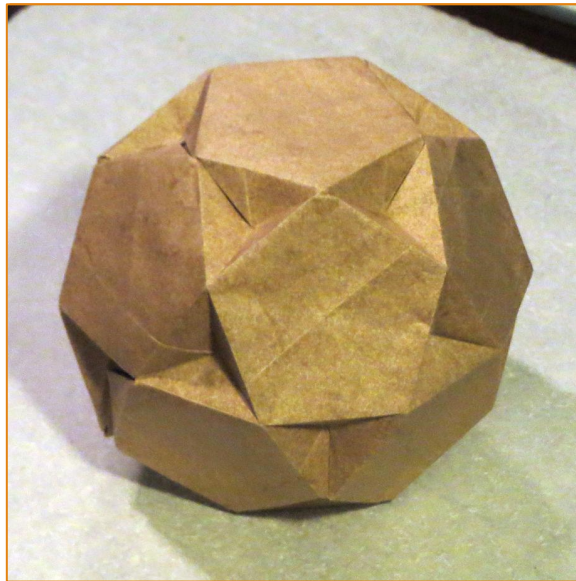
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Some tessellations use pentagons but do not have fivefold symmetry, such as this Cairo Tessellation.



# Single-Sheet Polyhedra

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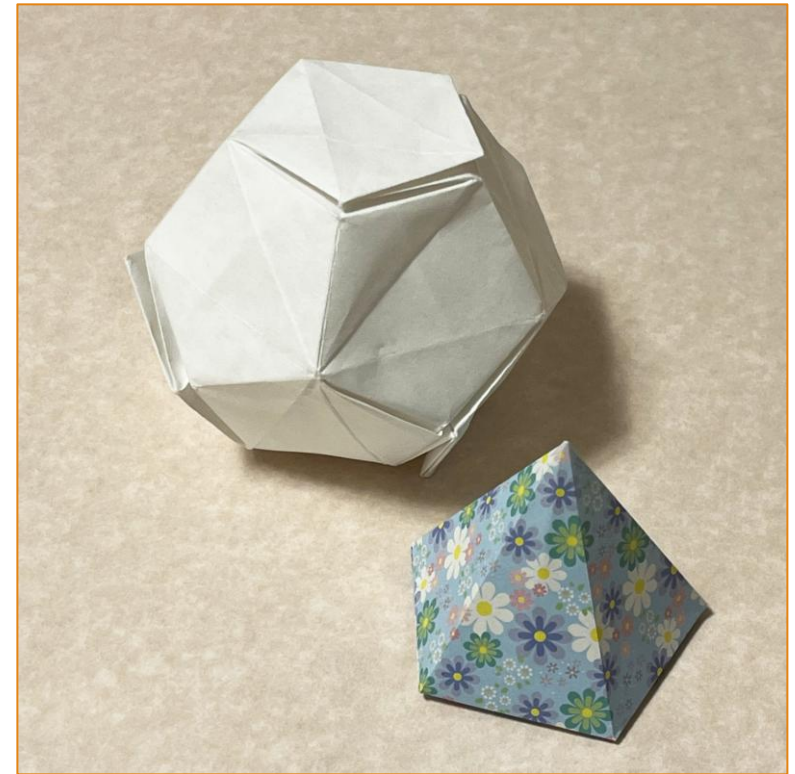
# Pyramid and Dodecahedron

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The Pentagonal Pyramid CP is latent in the pentagram.

The Dodecahedron is much harder.

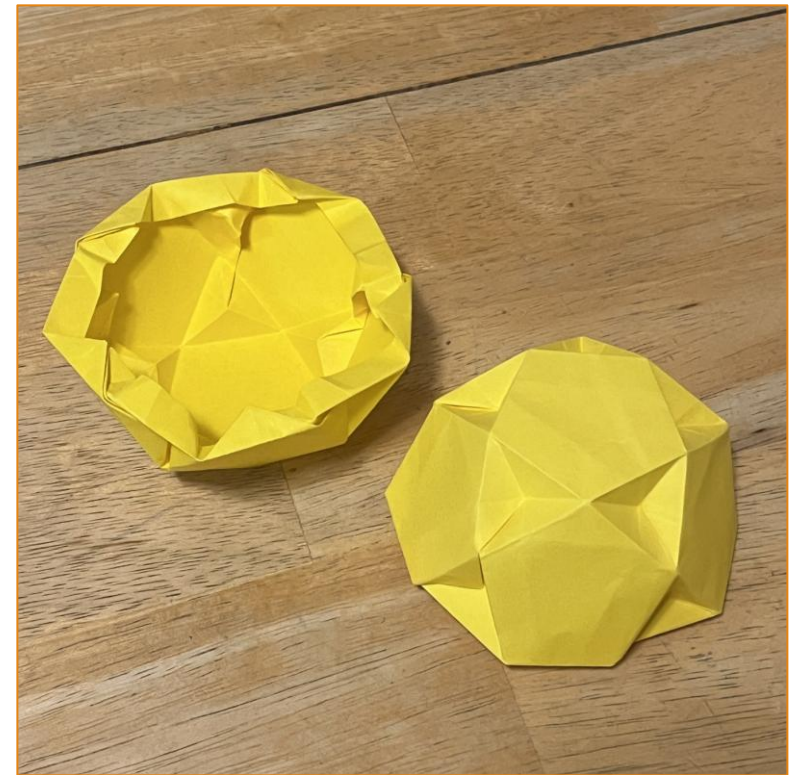
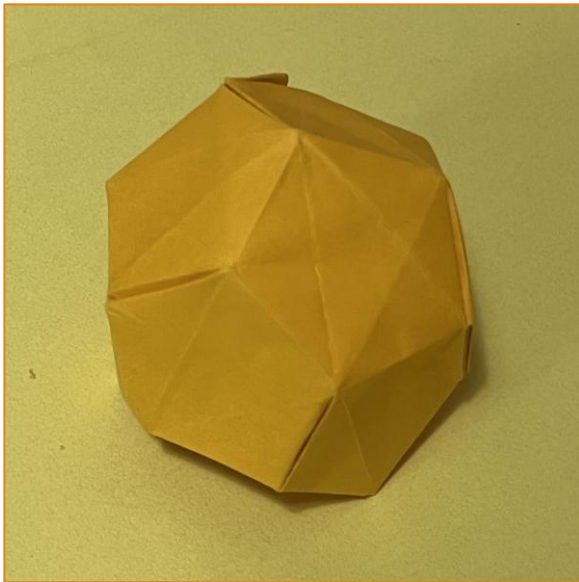
- There are many ways to fold a Dodecahedron, with different symmetries and layouts.
- The Dodecahedron can be a base for many other models. The best layout depends on the use.



# Domes and Bowls

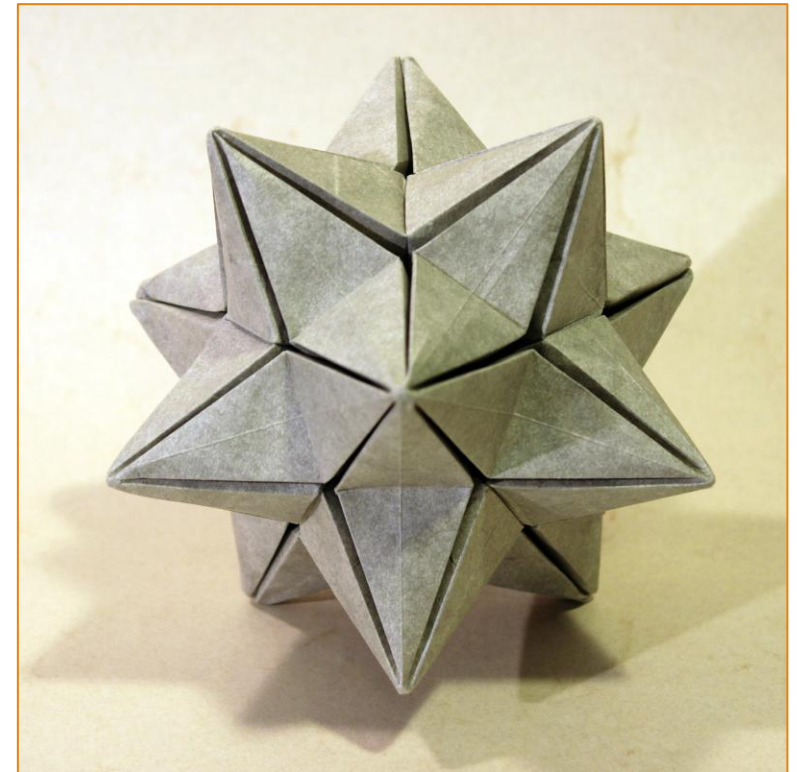
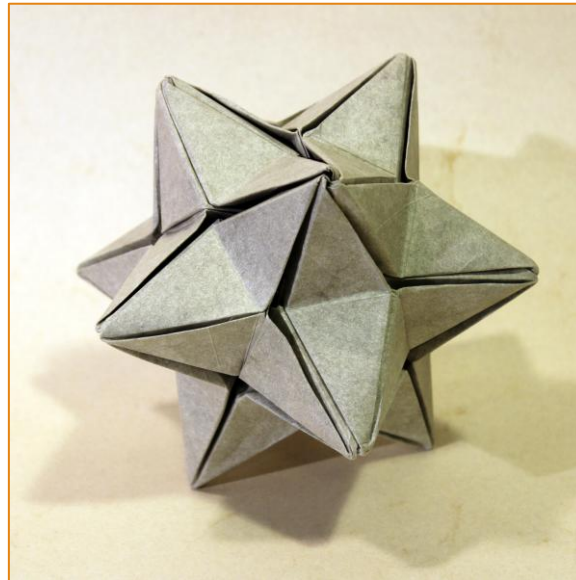
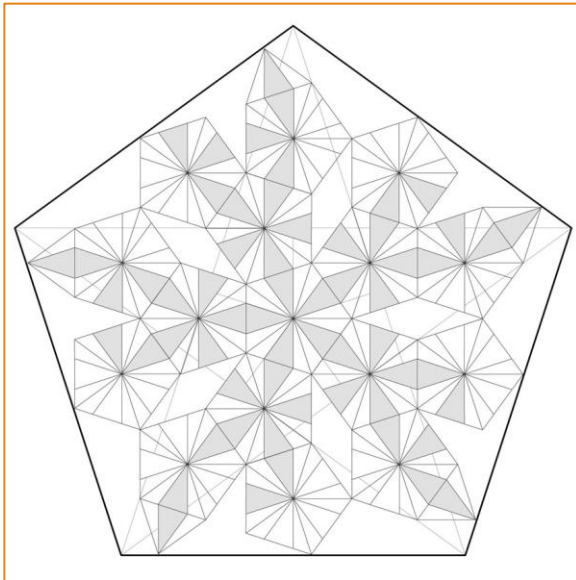
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Can be used studies for models with complicated geometries, or complete models in their own right.



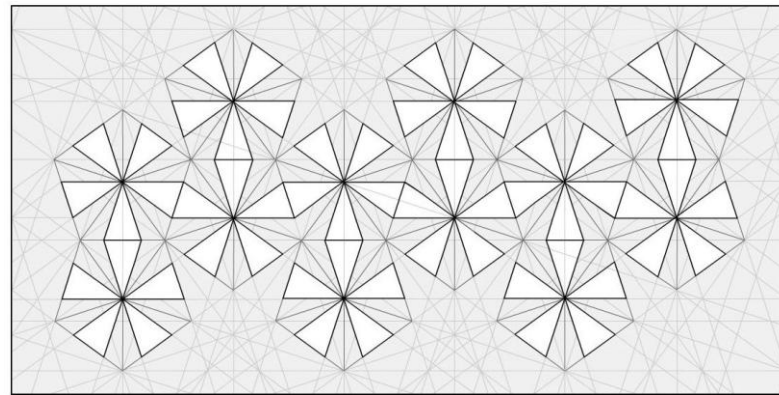
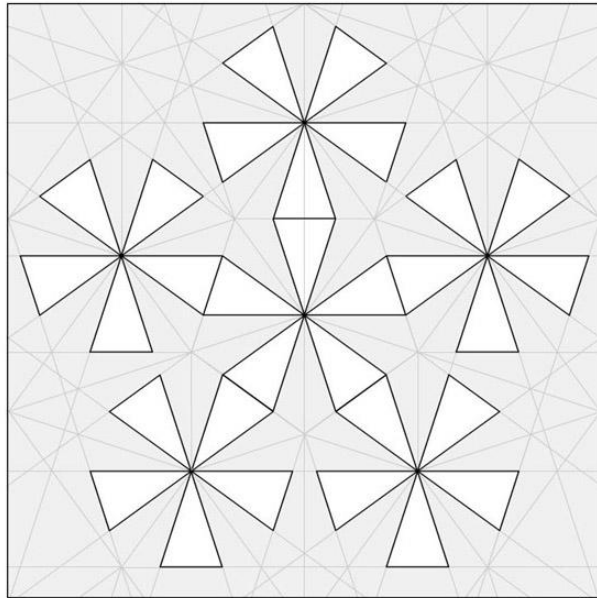
# Stellated Dodecahedron

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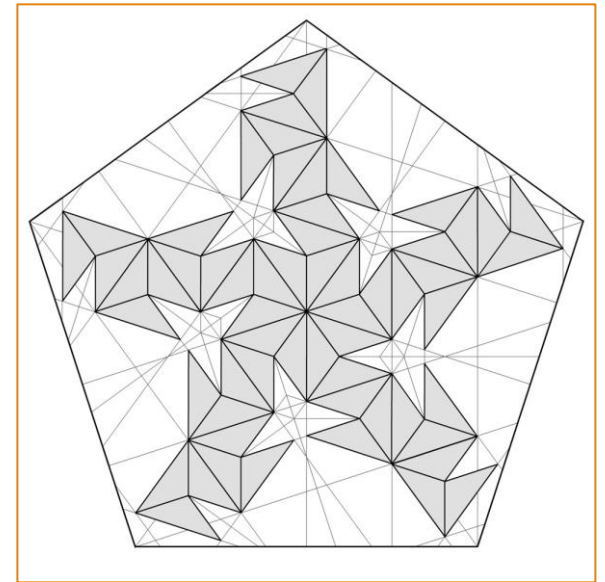
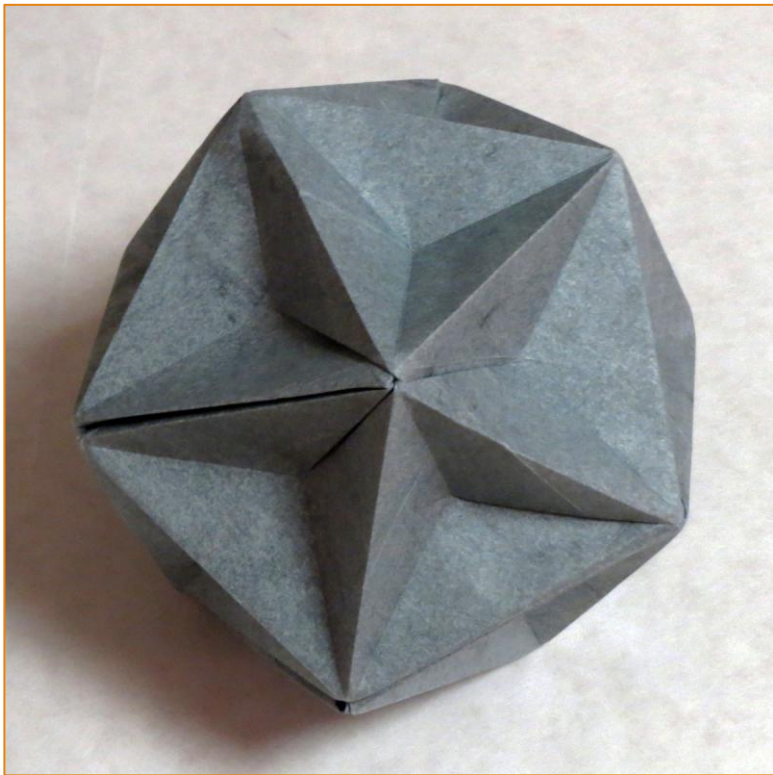
# Stellated Dodecahedron - Studies

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# Great Dodecahedron

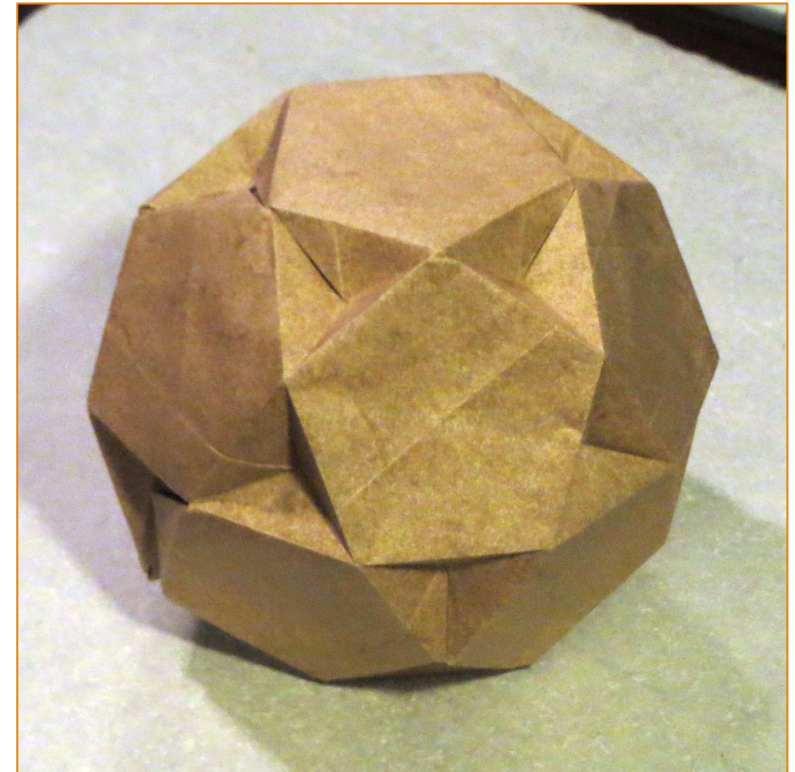
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# Dimpled Dodecahedron

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Dual of the Dimpled Icosahedron, a very challenging shape to fold.



# Aside – Icosahedral Symmetry

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Folded with a triangular or hexagonal grid, but can suggest pentagonal symmetry in polyhedra.



# Tessellated Polyhedra

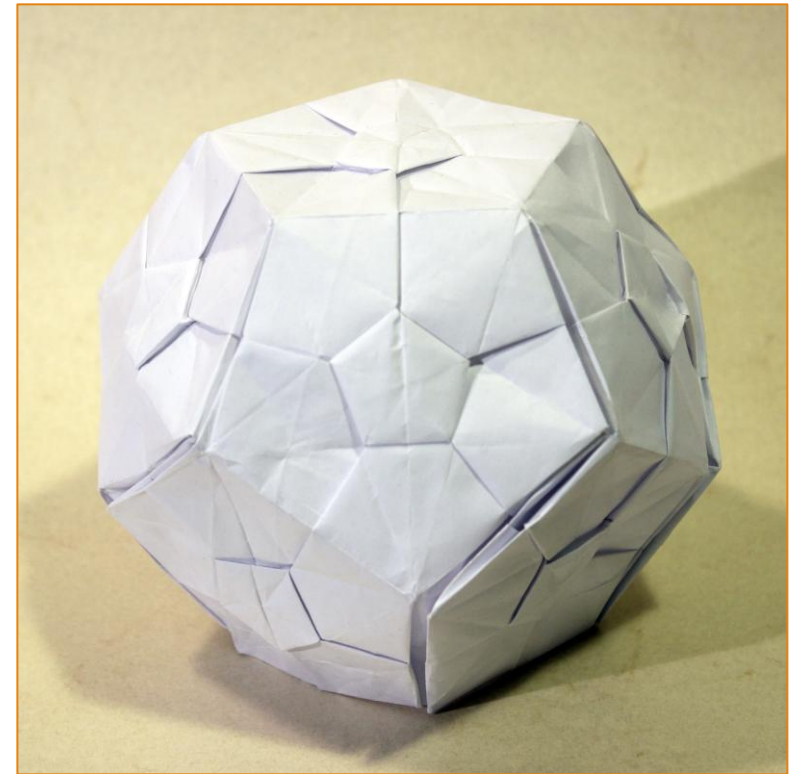
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Developed from Fractal Tessellations

A fun alternative to modulars

First embed the tessellation in the right places in the sheet

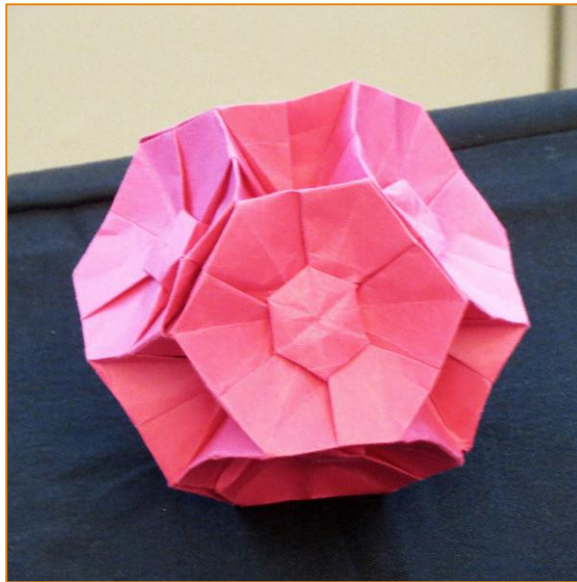
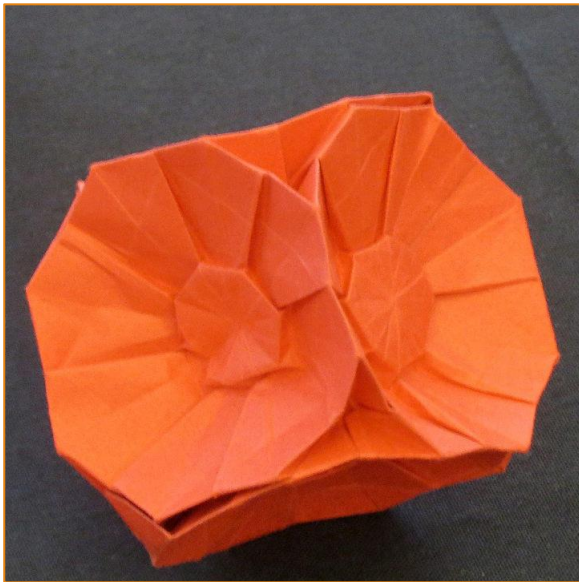
Then take that and create the 3-D form



# Flowerballs

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Can be made with multiple symmetries using the same principle.



# New Directions

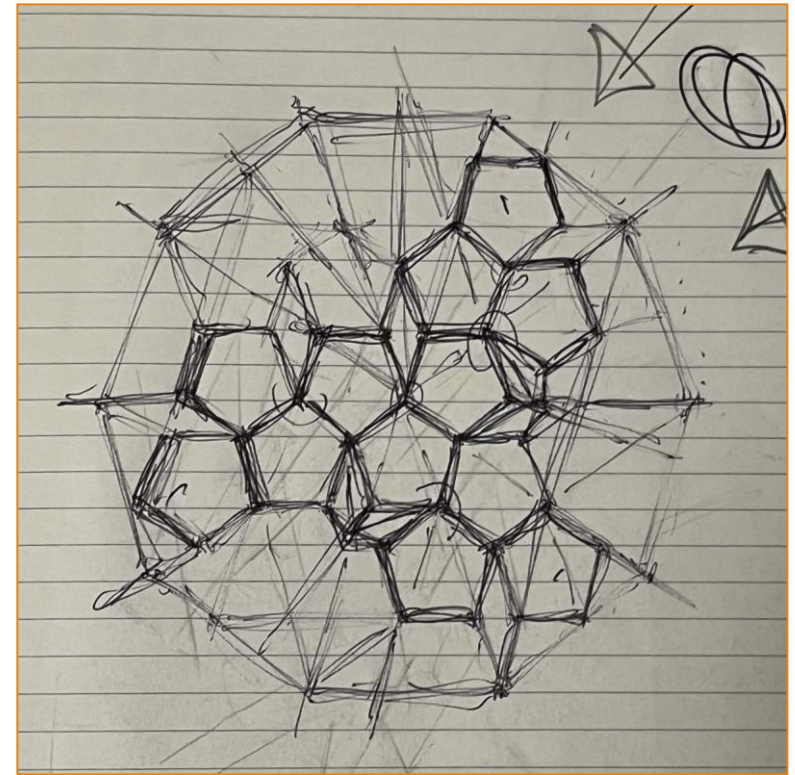
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# Vertex-Centered Dodecahedron

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Folded from a decagon; three vertices come together at the close.



# Vertex-Centered Dimpled Dodecahedron

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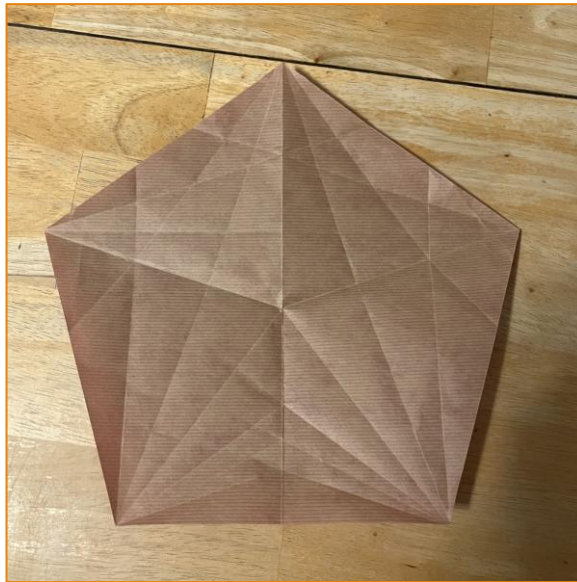
Very hard to close.



# Star People

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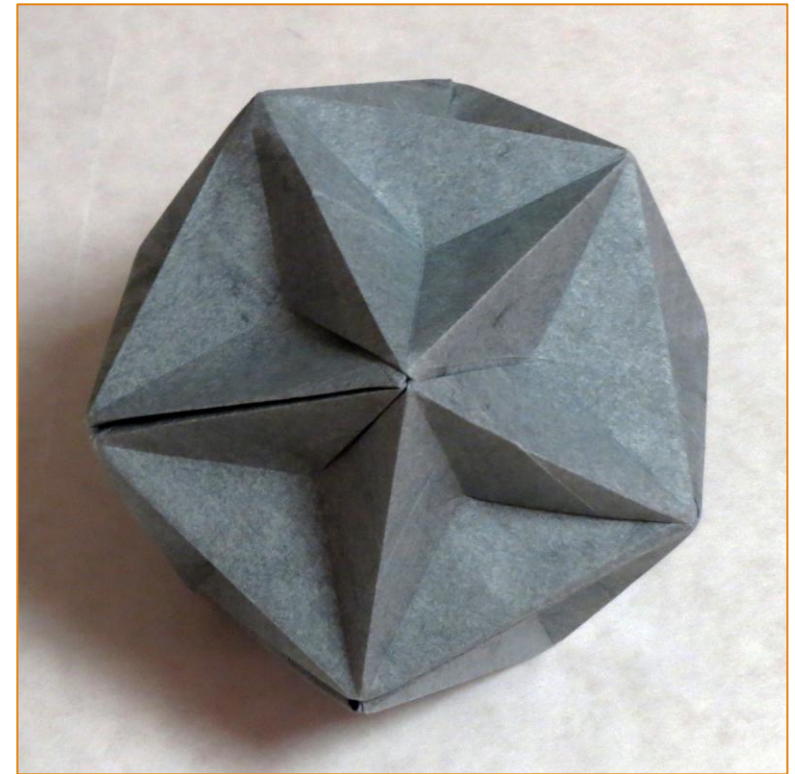
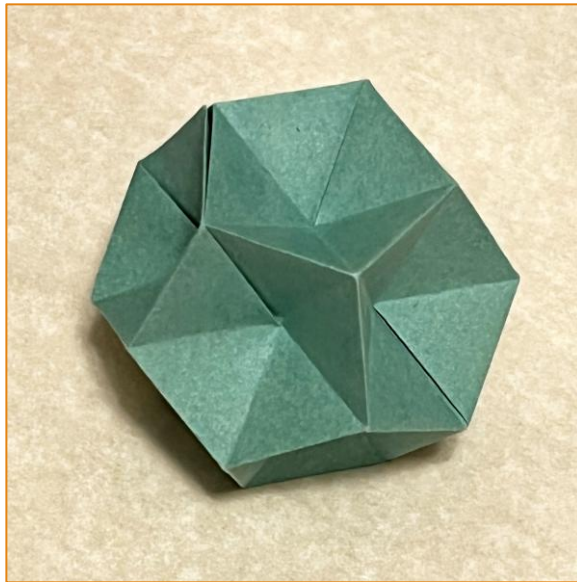
Human figure base from a pentagon



# Star Heads

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Human face base from a pentagon



# Discussion

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Any questions?

