

Explore Math with Origami

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The diagrams in this book were produced with Macromedia's Freehand, and image processing was done with Adobe Photoshop. The Backtalk family of typefaces was used for the body text. Futura was used for the cover and the headers.

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Introduction

For too many learners, mathematics is a jumble of rules and formulas designed for passive consumption. One of our favorite textbook authors, Harold Jacobs, challenges this notion by subtling his geometry book, *Seeing, Doing, and Understanding*. These three words informed our goals for this collection. To “see” math is to make abstract concepts accessible by recasting them in visual terms. To “do” math requires active engagement, treating the subject like a science course where you perform experiments. And to “understand” math is to make sense of what you notice and wonder as you investigate.

Here we offer the ancient art of origami as a unifying framework for bringing all three aspects of math to life. Origami is a popular pastime where squares, and sometimes other shapes, are folded into animals, human figures, and decorative designs. What may not be apparent is that even simple origami models contain deep mathematical principles baked into their folding sequences. Experts in the field of origami math have used origami to design compact foldable kayaks, shape-shifting surgical robots, and an elegant folding structure for the James Webb Space Telescope.

Geometry is the focus of this collection. You’ll learn about triangles, parallelograms, and rhombuses as you fold them from squares. You’ll construct the classic Tangram shapes and use them to create and solve puzzles. With the help of classmates and friends, you’ll make pentagonal tiles that fit together in surprising ways to cover a floor. And you’ll sample other areas of mathematics, including statistics, probability, and exponential growth.

Origami directions are typically quite specific, with landmarks indicating where to place the creases. But in many of these projects, you are free to pick the locations of your folds. These choices help to convey the generality of mathematical results. “The angles of any triangle sum to 180° ” may sound ho-hum, but if you and several friends each fold a random triangle and then compare the sums of their angles, it will likely be far more memorable – and amazing – to learn that your results are all the same.

We hope that this collection leaves you eager to delve deeper into the discoveries you make. The geometry textbooks cited in Recommended Reading are excellent resources for a more formal, proof-based introduction to the math.

So, grab some paper, start folding, and explore math with origami!