# Number Lines of Twists and Design Equations for Tessellations 

Rectas Numéricas de Giros y Ecuaciones de Diseño para Teselados

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## Tessellations are made of tiles



## General Triangle Twists / Giros Triangulares Generalizados

Triangle twists can be transitioned from one size to the next by shifting one pleat one grid spacing over.

This can be used to "count up" and make bigger and bigger twists (black arrows).
It can also be used to "count down" and find the connection with twists on the other side of the paper (green arrows).


## Other Shapes / Otras Formas



## Hybrid Twists / Giros Híbridos




## Way too many twists / Demasiados giros

So not only are there an infinite number of each shape of twist, but squares and rhombi have two dimensions of hybrids and hexagons have three dimensions of hybrids.

This gives us lots of options for things to fold, but it's also an intimidating space to search if we don't know what we're looking for.

Let's see if we can narrow down the options with some equations!

## Design Equations: Alternating and Mirrored / Alternado y Reflejado

For several tilings, you can pick two twists and a spacing to define a tessellation:

- Equilateral Triangles, Right Triangles Slope, 8 and 4 Right Triangles
- Squares, Aligned Rhombi, Trapezoids
- Hexagons and Triangles 6-fold (HT6)

With others (including all of the above except HT6), you can pick the contents of a column or square/triangular block and then mirror it:

- Small Squares and Right Triangles, Large Squares and Right Triangles, Split Rhombi, Hexagons and Triangles 2-fold (HT2), Hexagons and Triangles 3-fold, Mirror Chevron Rhombi, Split HT2, Star Rhombi and 6 Triangles, Trapezoids and 6 Triangles, etc


## Extended Triangle Symmetries

Triangle tilings are the easiest tiling to fold extendedsymmetry designs on.

1. Pick some subset of the tiling that makes a triangle (like four triangle tiles).
2. Make sure that your choices in that block are 3fold rotationally symmetric (inner/outer twists)
3. Mirror, alternate, or match with another block that fits with your block's pleats

Other options are also possible, but not fully described.


## Square Symmetries

In order to have a position of 4 -fold rotational symmetry in a repeating pattern, you must also have a second position of 4 -fold rotational symmetry and a position of 2-fold rotational symmetry.

For alternating patterns on squares, one twist (right-handed) occupies all the blue-dot 4-fold positions and the other twist (left-handed) occupies all the gold-dot 4-fold positions. The 2fold positions are in the center of each loop of four twists.

| S 4 | S 4 | S 4 | S 4 | S 4 |
| :--- | :--- | :--- | :--- | :--- |
| S 4 | S 4 | S 4 | S 4 | S 4 |
| S 4 | S 4 | S 4 | S 4 | S 4 |
| S 4 | S 4 | S 4 | S 4 | S 4 |
| S 4 | S 4 | S 4 | S 4 | S 4 |

First Square Symmetry Extension



## Second Square Symmetry Extension





S4 $-\mathrm{S} 4=\mathrm{H} 2$ (34-(3) $=-2 *+2($ (32 - (34) $)$



## More Options

These aren't the only square symmetry expansions either - you can keep putting those two 4 -fold positions further and further apart as long as the 2-fold position lands either on a twist or in the middle of a loop of four twists.

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## Hexagon Symmetries

In order to have a position of 6 -fold rotational symmetry in a repeating pattern, you must also have positions of 3 -fold and 2 -fold rotational symmetry.

For alternating tessellations on HT6 tilings, the 6 -fold position is in the center of each hexagon and the 3 -fold position is in the center of each triangle. The 2 -fold positions are in the center of each loop of four twists.


## First Hexagon Symmetry Extension




## Second Hexagon Symmetry Extension


(13) - (자)= (ㄷ) - (ㅗ


## More Options

Just like on the square tiling, you can keep extending the hexagon symmetry as much as you want, as long as you find suitable locations for the 2 -fold and 3 -fold positions.

You can also modify these symmetries by adding spacing on select lines or by using mixed-depth twists.

Symmetry can also be sacrificed by adding spacing in only one direction or by placing a hybrid twist in the center.

As long as the hexagon and triangles are shifted in the same direction on their number lines, you can shift just a small portion of the pattern independently of the rest.





## Not Just on the HT6 Tiling





# Questions? Preguntas? 

