# Mafra Rings – a Fallen Paradigm


When you break (successfully) a rule not subject to discussion an interesting artistic result appears.

Any geometric model must follow the rules of an exact mathematical construction, especially the construction of angles. Looks like an absolute truth, isn't it? And it often leads to complex folding process leaving unnecessary creases.

What if this paradigm of geometric folded is abandoned and exactness is be traded for simplicity. Series of more than 100 simple models called Mafra Rings resulted in such a way. Modules for all of the rings share similar simple crease pattern (5 lines, 4 folds). Rings may contain from 5 to 18 modules (including such strange numbers like 7, 11, 13). Actually none of the rings is mathematically perfect (a mathematician would say that they do not exist), but all of them close easily.

Hopefully Mafra Rings open a way for other geometric models that are simple to fold at the cost of the absolute mathematical exactness of the construction.


Have you ever thought why stars and rings of 8 or 6 modules are so popular in origami while of 7 or 9 modules are not? The answer is simple. To have a ring of 8 modules you must find reference points that form an angle of 45° (i.e. 1/8 of the full angle) to position subsequent modules. What is easy. It is also easy to construct an angle of 60° (1/6 of the full angle) for a ring of 6 modules. But an angle of 40° (1/9 of the full angle) is much harder to construct. Such angle is not constructible with ruler and compass. It is constructible in origami, but requires a fold "two points onto two lines" that leads a to complex folding sequence. Looks hopeless. Even if we find a sequence, it will require mane creases and a model will look ugly. And who will enjoy folding a complex ring of nine modules?

To come out of the cul-de-sac we must abandon the "obvious" paradigm:

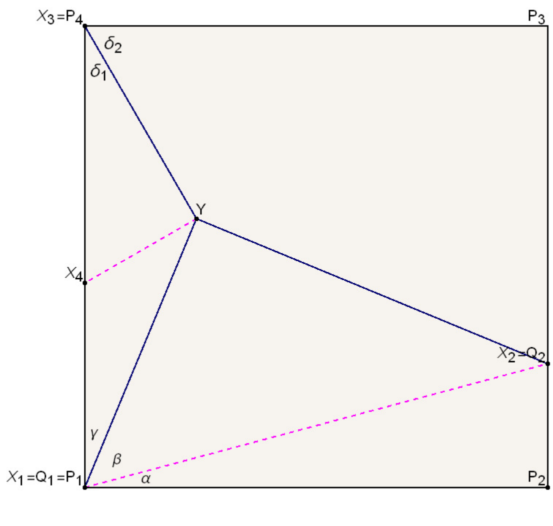
*Geometric models require mathematically exact construction.*

And assume another:

*Find an easy, but enough accurate construction.*

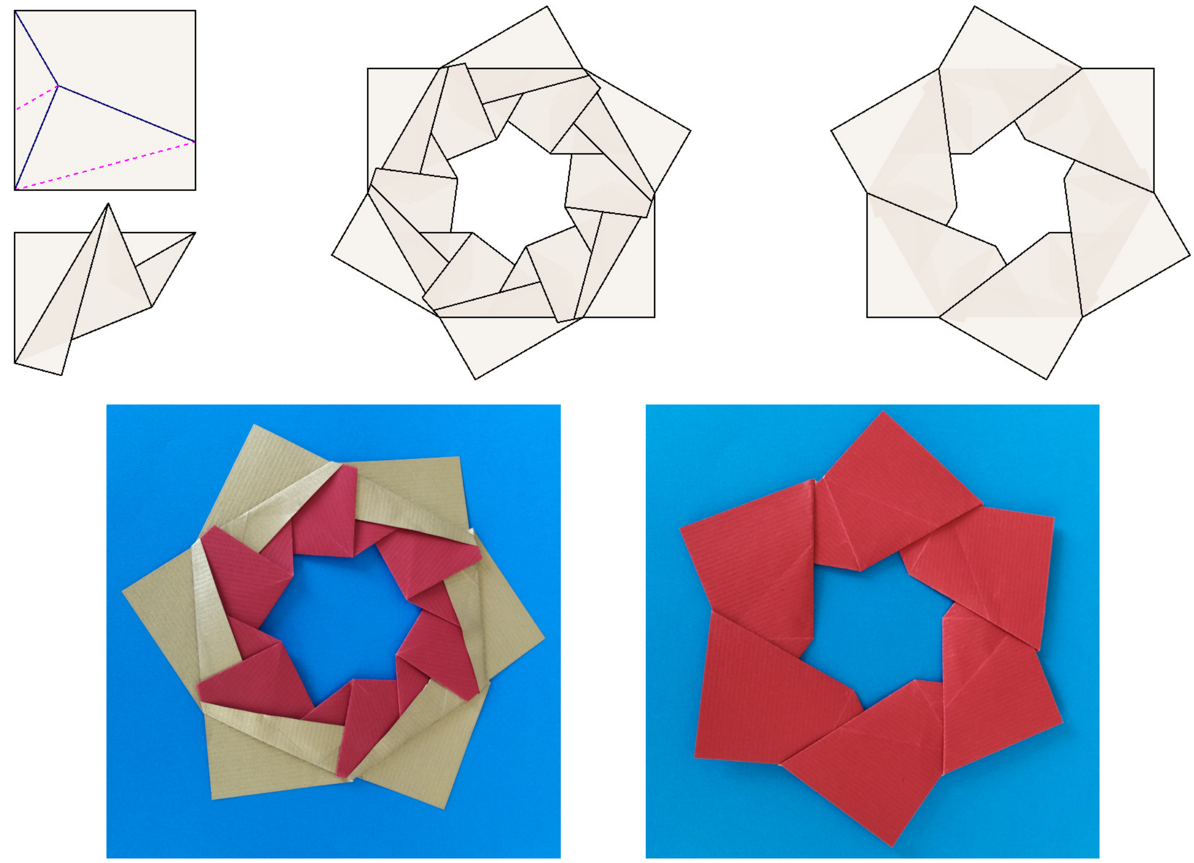
Of course we must decide what does it mean "easy" and "enough accurate". Mafra Rings show how such principle may be realized to find variety of easy models with nice effect/complexity ratio.

A module for a Mafra ring is based on the Versailles module designed by Krystyna Burczyk. Each module may be described by three angles α, γ, $\delta_1$ that fully determine a crease patter for the module.



The folding sequence for a module consists of 5 simple steps:

1. Fold an angle $\alpha$. Denote the intersection with a raw edge as $X_2$
2. Fold an angle $\gamma$.
3. Fold an angle $\delta_1$ . Denote the intersection with the angle $\gamma$ as Y.
4. Fold a line between points Y and $X_2$.
5. Fold a rabbit ear using three lines coming out from the point Y, A new fold between Y and $X_4$ appears.



Example of Mafra module for angles $\alpha$=15°, $\gamma$=22.5°, $\delta_1$=30°.
Crease pattern, module and closed ring (both sides), folded ring.

Now we can define “easy” and “enough accurate”.

The “enough accurate” condition was established empirically by folding generated modules. Finally it was set as: the total rotation angle for a ring cannot excess the full angle more than 8° and underflow more than 4°. Such an error is practically invisible (it’s about 0.5° per module) and any mathematically accurate method will introduce larger error coming fom natural errors of folding.

To define what is easy first we notice what is not. In our case, easy folding sequence cannot require:

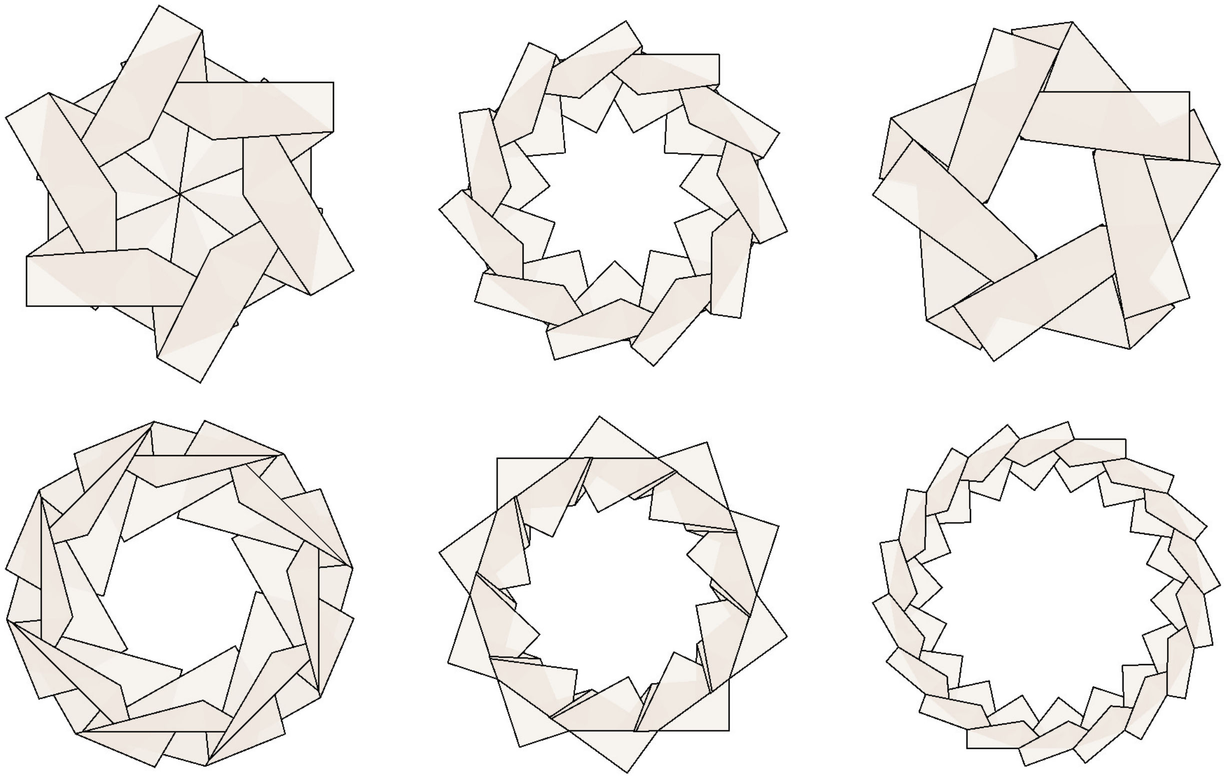
1. Measure angles or distances.
2. Too many steps (for the construction of a desired angle).
3. Too small angles.
4. Too short flap used to lock the modules in a ring.

What could be stated as easy folds:

1. An angle bisector (a line onto a line, divides an angle into halves)
2. A segment bisector (a point onto a point, divides a segment into halves).
3. Division of an edge into 3 and 5 equal parts (Haga theorem or Fujimoto approximation method).
4. Folding a corner onto a vertical line while a crease line starts at the adjacent corner (a fold similar to the division of a right angle into angle of 30° and 60°).

Finally any angle smaller than 15° and any flap shorter than 10% of side length are disqualified as not easy (arbitrary selection based on the folding experience).

The rest was applying brute force to check all possibilities of easy angles and select combination that lead to foldable models. A program in Mathematica using Tessellatica package by R. J. Lang was written to perform such task. And 163 easy rings were found covering all possible numbers of modules in a range from 5 modules to 18 modules. Morover the appearance varies more that I initially expected.



Examples of Mafra rings

Not all of the designs are actually folded at the moment, but so far every virtual model can be folded with physical paper and produces expected result. Average folding time is about 10 minutes (template, modules and assembly).

Looks that trading mathematical accuracy for simple folding sequence with sufficient accuracy may lead to appealing designs in the field of geometric folding.